

5 hard questions - part 1



Non Calculator

1. Simplify fully $\frac{2x-12}{x-4} \div \frac{2x^2-6x-36}{x^2-9x+20}$ ←

(4 marks)

$$\frac{\cancel{2(x-6)}}{\cancel{x-4}} \times \frac{\cancel{(x-4)}(x-5)}{\cancel{2(x-6)}(x+3)}$$

$$\frac{x-5}{x+3}$$

$$\begin{array}{r} -72 \\ / \quad | \\ -12 \quad +6 \end{array}$$

$$\underline{2x^2 + 6x - 12x - 36}$$

$$2x(x+3) - 12(x+3)$$

$$(2x-12)(x+3)$$

$$2(x-6)(x+3)$$

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$$-2y = -x + 10$$

$$y = \frac{1}{2}x - 5$$

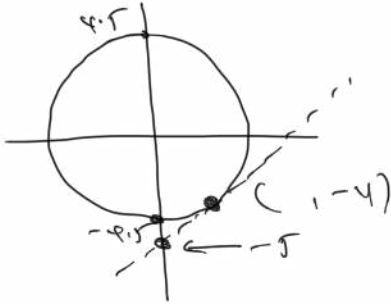


Non Calculator

2. Prove algebraically that the straight line with the equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$ $\leftarrow r^2 = \sqrt{20}$ (4 marks)

$$x = 2y + 10$$

only one solution



$$\begin{aligned} x^2 + y^2 &= 20 \\ (2y + 10)^2 + y^2 &= 20 \\ 4y^2 + 40y + 100 + y^2 &= 20 \\ 5y^2 + 40y + 80 &= 0 \\ y^2 + 8y + 16 &= 0 \\ (y + 4)(y + 4) &= 0 \\ \therefore y &= -4 \end{aligned}$$

ONE solution

$$\begin{aligned} \therefore x &= 2 \\ y &= -4 \end{aligned}$$

Substitution

$$\begin{aligned} x &= 2y + 10 \\ &= 2(-4) + 10 \\ x &= 2 \end{aligned}$$

3. The n th term of a sequence is $n^2 + 2n + 1$

Two consecutive terms in the sequence have a difference of 13.

Work out the two terms.

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(3 marks)

$$\begin{aligned} 1^{\text{st}} & (1)^2 + 2(1) + 1 = 4 \\ 2^{\text{nd}} & (2)^2 + 2(2) + 1 = 9 \\ 3^{\text{rd}} & (3)^2 + 2(3) + 1 = 16 \\ 4^{\text{th}} & (4)^2 + 2(4) + 1 = 25 \\ 5^{\text{th}} & (5)^2 + 2(5) + 1 = 36 \\ 6^{\text{th}} & (6)^2 + 2(6) + 1 = 49 \end{aligned}$$

set = 6th term

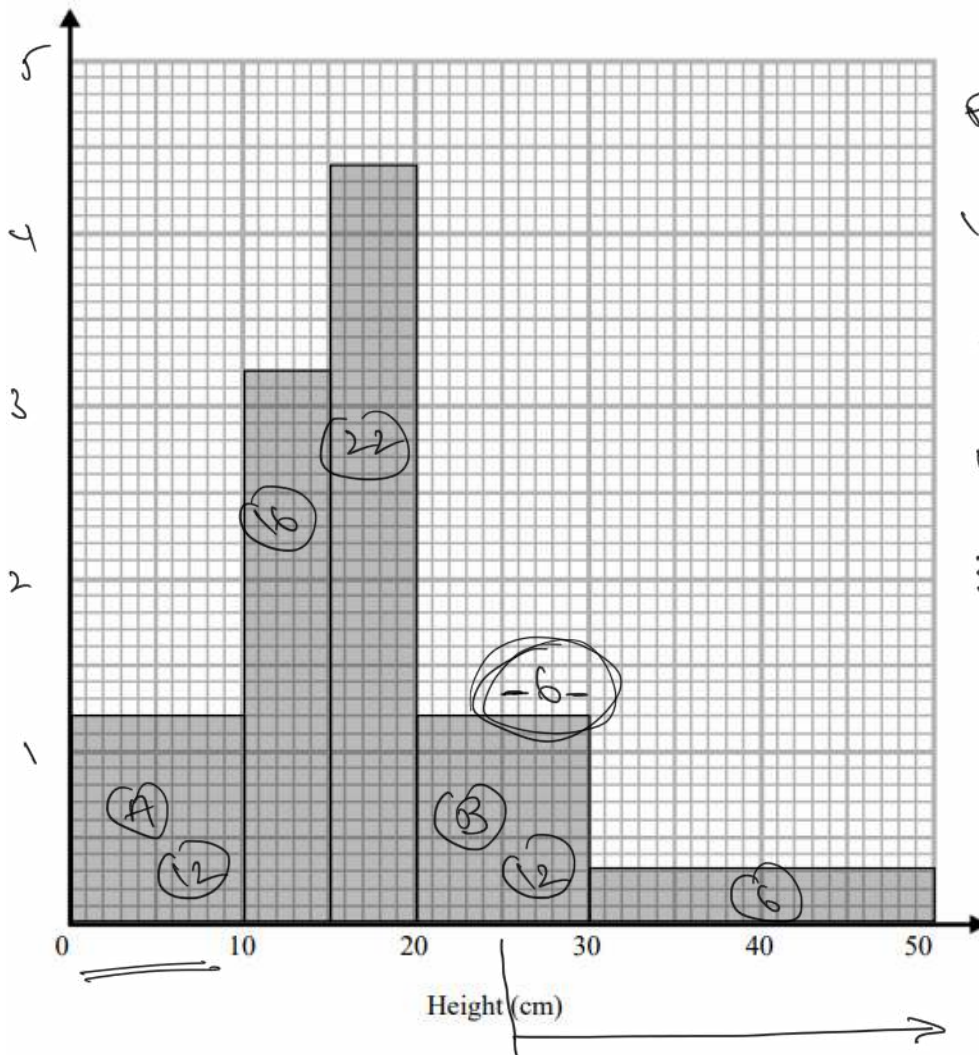
(DIB B B)

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Non Calculator

4. The histogram shows information about the height of some plants.



FD = $\frac{1}{2} \times \frac{1}{2}$
 $\therefore 2 = \frac{1}{2} \times \frac{1}{2}$
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(a) Work out an estimate for the number of plants over 25cm tall. (3 marks)

$$\begin{aligned} \text{Total plants} &= 12 + 16 + 22 + 12 + 6 \\ &= 68 \\ \text{Over 25cm} &= 6 + 6 = 12 \\ \therefore \frac{12}{68} &= \frac{3}{17} \end{aligned}$$

(b) Explain why your answer to part (a) is an estimate (1 mark)

'Grouped data' - we don't know how many plants are between 25-30cm

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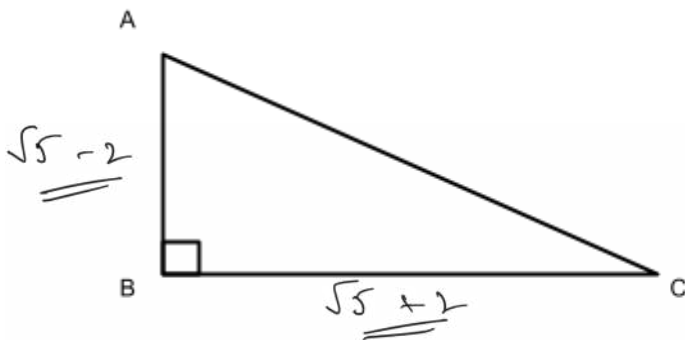
Non Calculator

5. The diagram shows a right angle triangle.

$$AB = \sqrt{5} - 2$$

$$BC = \sqrt{5} + 2$$

All measurements are in m.



Work out, leaving your answers in surd form where appropriate,

(a) the area of the triangle

(2 marks)

$$\frac{1}{2} (\text{base} \times \text{height})$$

$$\frac{1}{2} (\sqrt{5} + 2)(\sqrt{5} - 2)$$

$$\frac{1}{2} (5 - 4) = \frac{1}{2} (1) = \underline{\underline{\frac{1}{2} \text{ m}^2}}$$

(b) the length AC 'in its simplest form'

(3 marks)

$$a^2 = b^2 + c^2$$

$$= (\sqrt{5} - 2)(\sqrt{5} - 2) + (\sqrt{5} + 2)(\sqrt{5} + 2)$$

$$= 5 - 4\sqrt{5} + 4 + 5 + 4\sqrt{5} + 4$$

$$a^2 = 18$$

$$a = \sqrt{18}$$

$$= \sqrt{9 \times 2}$$

$$\underline{\underline{AC = 3\sqrt{2}}}$$