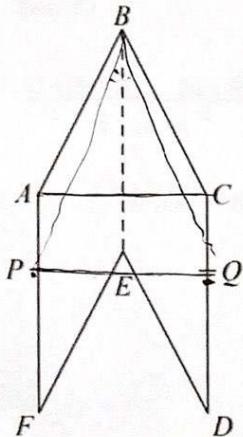




QT Tough Questions - Proving Cosine



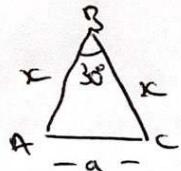
The diagram shows a hexagon ABCDEF.

ABEF and CBED are congruent parallelograms where $AB = BC = x \text{ cm}$.

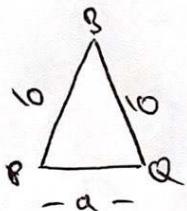
P is the point on AF and Q is the point on CD such that $BP = BQ = 10 \text{ cm}$.

Given that angle ABC = 30° , prove that

$$\cos PBQ = 1 - \frac{(2-\sqrt{3})}{200}x^2$$



$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ &= x^2 + x^2 - 2(x)(x)\left(\frac{\sqrt{3}}{2}\right) \\ a^2 &= 2x^2 - x^2\sqrt{3} \end{aligned}$$



$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \quad / \cos PBQ \\ &= 10^2 + 10^2 - 2(10)(10)(\cos A) \\ a^2 &= 200 - 200 \cos A \end{aligned}$$

$$2x^2 - x^2\sqrt{3} = 200 - 200 \cos A$$

$$200 \cos A = 200 - 2x^2 + x^2\sqrt{3}$$

$$200 \cos A = 200 - x^2(2 - \sqrt{3})$$

$$\cos A = \frac{200 - x^2(2 - \sqrt{3})}{200}$$

$$\cos A = \frac{200 - x^2(2-\sqrt{3})}{200}$$

$$\cos A = \frac{200}{200} - \frac{x^2(2-\sqrt{3})}{200}$$

$$\cos A = 1 - \frac{(2-\sqrt{3})x^2}{200}$$

$A = \underline{\text{Angle PBQ}}$

From right angle at O
 $\angle QOB = 90^\circ - \theta$
 $\angle POB = 60^\circ - \theta$



From right angle at O
 $\angle QOB = 90^\circ - \theta$
 $\angle POB = 60^\circ - \theta$



$\tan(90^\circ - \theta) = \frac{PB}{OB}$

$PB = OB \cdot \tan(90^\circ - \theta)$ = Focal

$(\text{Focal})^{1/2} = OB \cdot \tan(90^\circ - \theta)$

$\frac{(\text{Focal})^{1/2}}{OB} = \tan(90^\circ - \theta)$