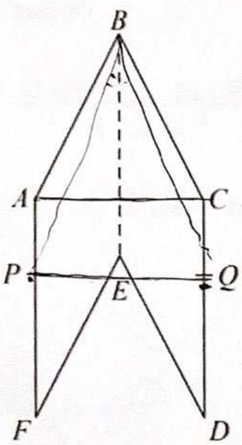
QT Tough Questions - Proving Cosine

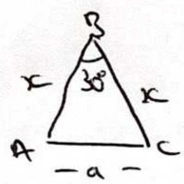


The diagram shows a hexagon ABCDEF.

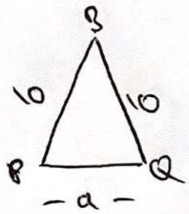
ABEF and CBED are congruent parallelograms where
AB = BC = x cm.
P is the point on AF and Q is the point on CD such that
BP = BQ = 10cm.

Given that angle ABC = 30^0, prove that

cos PBQ = $1 - \frac{(2-\sqrt{3})}{200}x^2$



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= x^2 + x^2 - \cancel{2}(x)(x)\left(\frac{\sqrt{3}}{\cancel{2}}\right)$$
$$a^2 = 2x^2 - x^2\sqrt{3}$$



$$a^2 = b^2 + c^2 - 2bc\cos A$$ — cos PBQ
$$= 10^2 + 10^2 - 2(10)(10)(\cos A)$$
$$a^2 = 200 - 200\cos A$$

$$2x^2 - x^2\sqrt{3} = 200 - 200\cos A$$
$$200\cos A = 200 - 2x^2 + x^2\sqrt{3}$$
$$200\cos A = 200 - x^2(2-\sqrt{3})$$
$$\cos A = \frac{200 - x^2(2-\sqrt{3})}{200}$$

$$\cos A = \frac{200 - x^2(2-\sqrt{3})}{200}$$

$$\cos A = \frac{200}{200} - \frac{x^2(2-\sqrt{3})}{200}$$

$$\cos A = 1 - \frac{(2-\sqrt{3})}{200}x^2$$

$A = \underline{\text{Angle}}\ PBQ$.